Online appendix to:
Entrepreneurship, Labor Market Mobility and the Role of Entrepreneurial Insurance

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A Facts about entrepreneurship

A.1 Fear of failure

Figure 1 plots the relation between the fear of failure index and the self-employment rate in the US as evidenced by the data collected by the Global Entrepreneurship Monitor. This relation is discussed in section 2.1 of the paper.

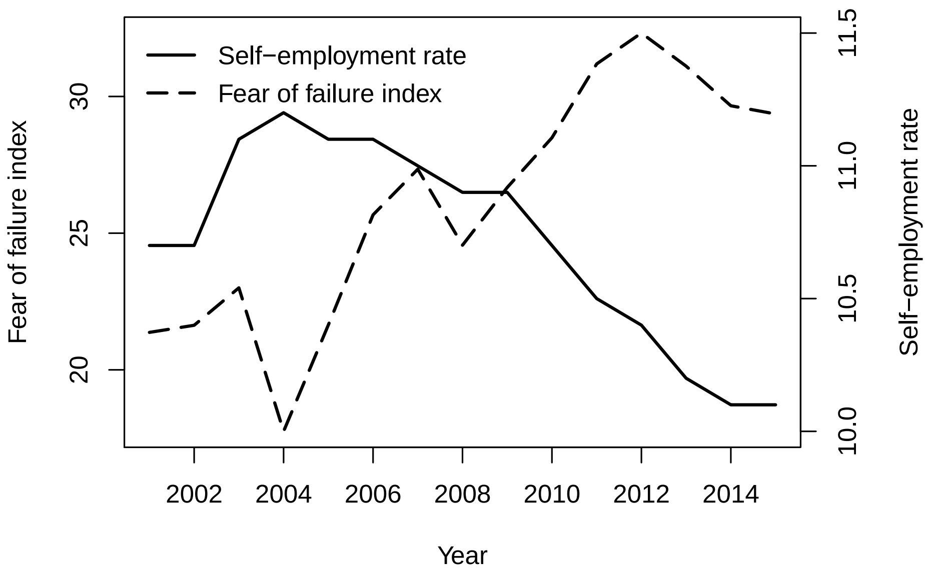

Figure 1. Fear of failure index and self-employment rate in the US. *Source:* Global Entrepreneurship Monitor and The Bureau of Labor Statistics (BLS). The Fear of Failure index measures the 18-64 old population perceiving good opportunities to start a business while indicating that the fear of failure would prevent them from doing so. The self-employment rate (over the working population) is the fraction of the 20-65 old population declaring themselves as self-employed (incorporated or not).

A.2 Additional occupational flows

Figures 2, 3, 4 and 5 below provide additional occupational flow evidence from the CPS in addition to the flow plot provided and discussed in section 2.2 of the paper. Figure 2 considers the period from 2001 to 2015 and Figure 3 considers the alternative data period 2012-2015 (excluding the 2009 - 2011 unemployment peak). Figure 4 only considers full-time occupation and discards part-time occupied households from the characterization. The shapes are quite similar to the one in the core paper. We notice however that the transition from paid-employment to entrepreneurship is *U-shaped* in the two dimensions of wage and educational attainment for these 3 figures. Finally, in figure 5 we display the yearly transitions. It appears that when considering only self-employed business-owners, the *S-shape* of the transition from entrepreneurship to paid-employment becomes a *hump-shape* decreasing only for *college* graduates. This could be due to the fact that large movements occur at a high frequency (quarterly frequency), suggesting that many lower than high-school individuals try to run a business but fail relatively quickly (before one year). We also did compute these transitions at a monthly frequency. Shapes are similar to those at a quarterly frequency.

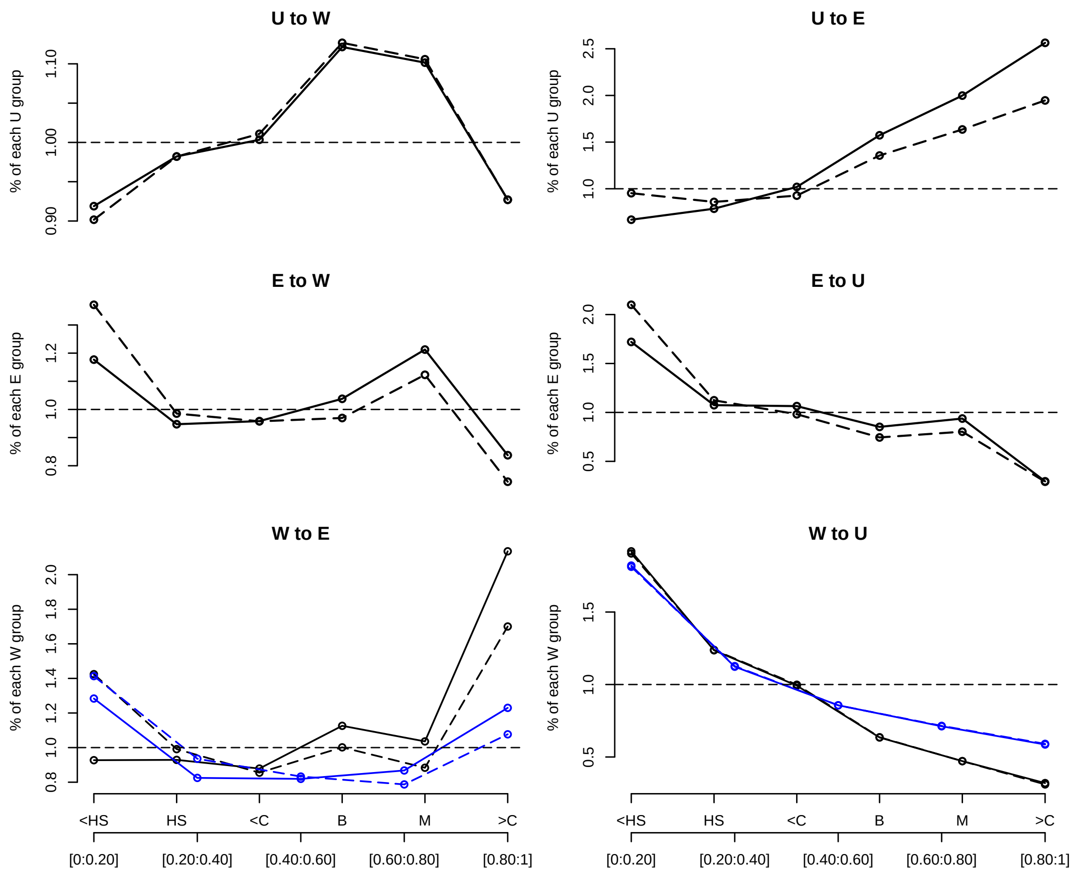

Figure 2. Quarterly flows from a given occupation to another by educational attainment (black) and wage earnings (blue), from 2001 to 2015. A dashed lines refer to self-employment only while a solid line to only self-employed business owners. *Data sources:* monthly basic CPS.

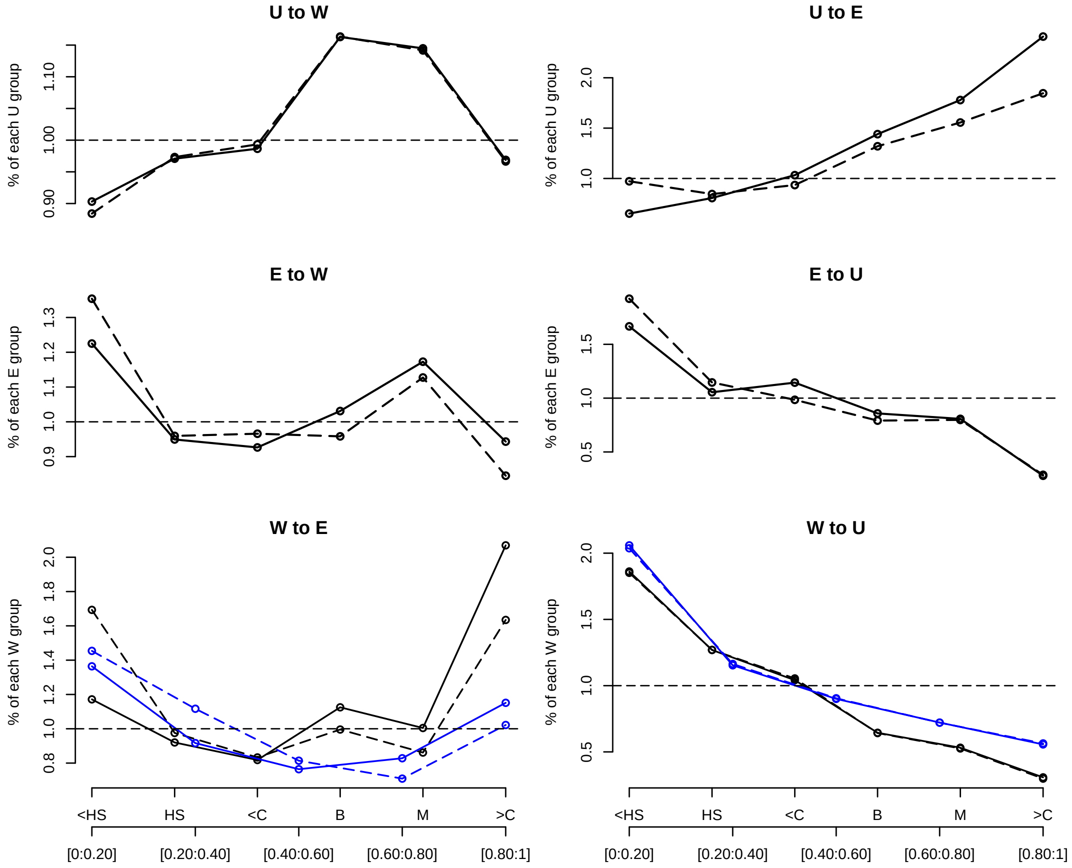

Figure 3. Quarterly flows from a given occupation to another by educational attainment (black) and wage earnings (blue), from 2012 to 2015. A dashed lines refer to self-employment only while a solid line to only self-employed business owners. *Data sources:* monthly basic CPS.

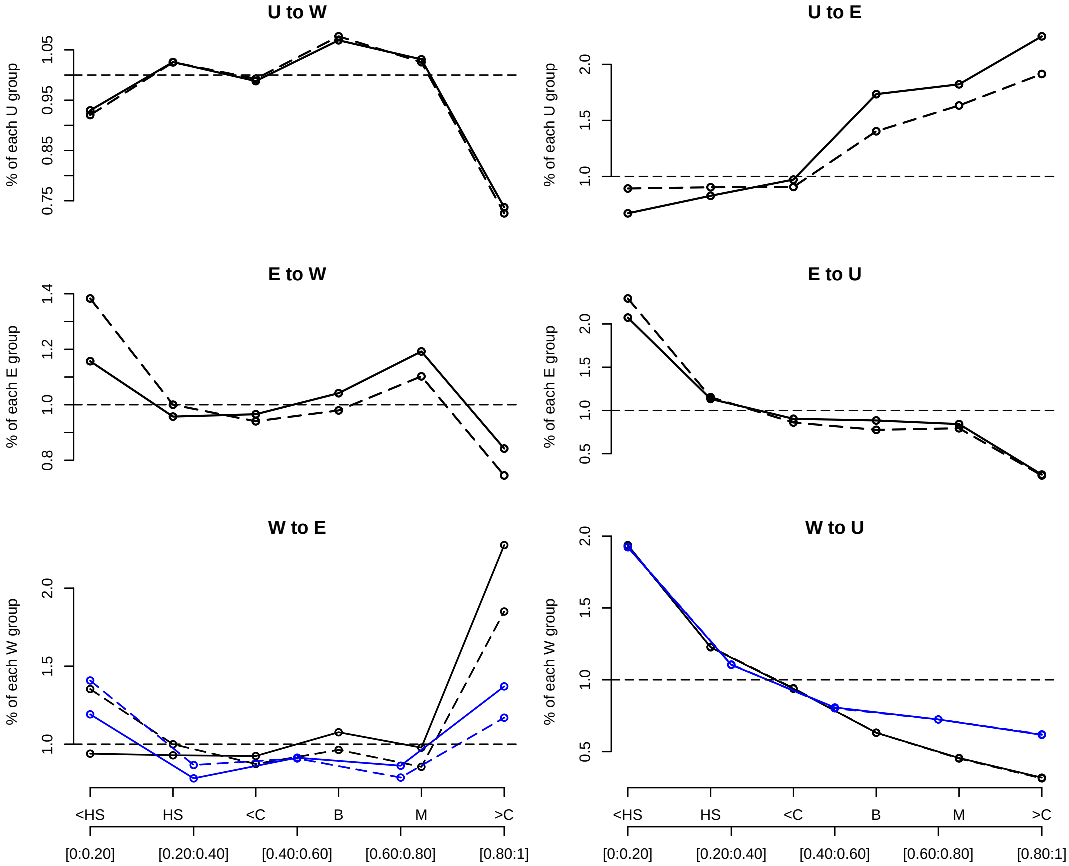

Figure 4. Quarterly flows from a given occupation to another by educational attainment (black) and wage earnings (blue), restricting to full-time movements. A dashed lines refer to self-employment only while a solid line to only self-employed business owners. *Data sources:* monthly basic CPS from 2001 to 2008.

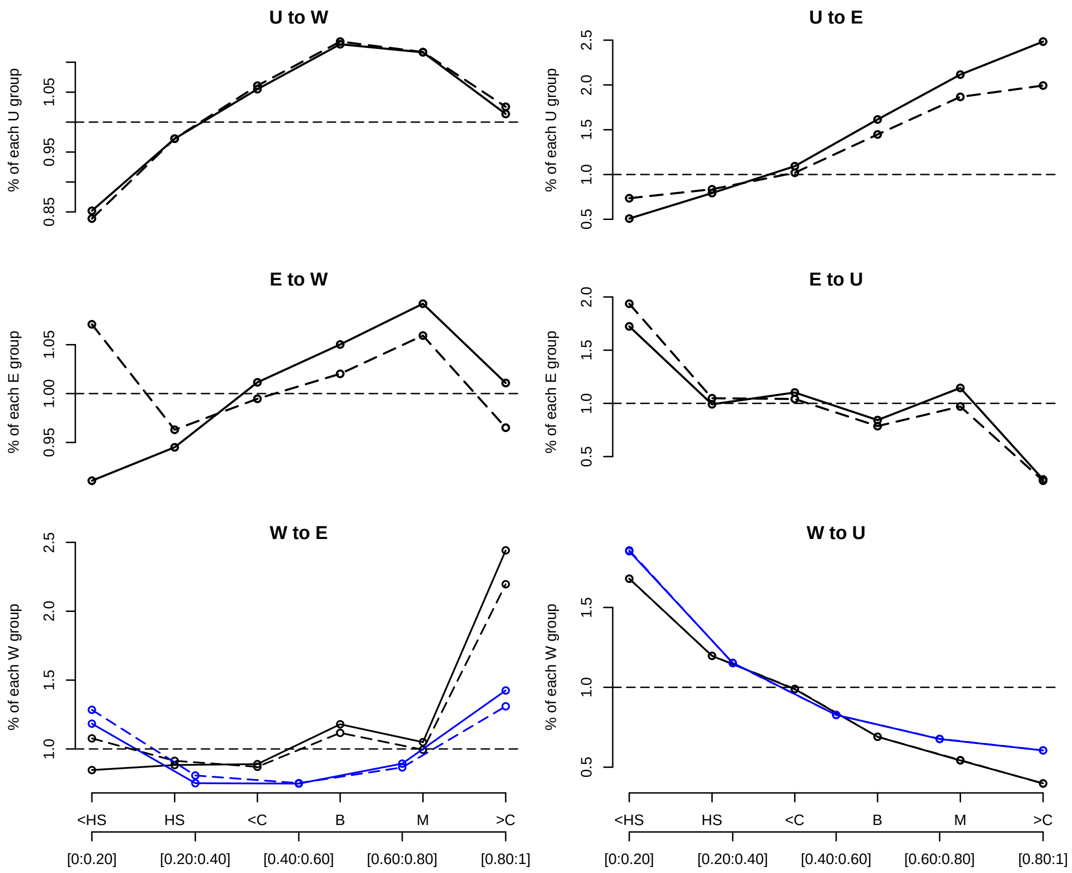

Figure 5. Yearly flows from a given occupation to another by educational attainment (black) and wage earnings (blue). A dashed lines refer to self-employment only while a solid line to only self-employed business owners. *Data sources:* monthly basic CPS from 2001 to 2015.

A.3 Yearly occupational flows

Using the CPS, we also compute the yearly occupational flows from 2001 to 2008. Table 1 summarizes the results. As a comparison, Cagetti and De Nardi (2006) obtain a yearly entrepreneurial exit rate toward paid-employment of about 23% within their model and a yearly worker exit rate toward entrepreneurship of about 2.4%. We find quite similar flows using our definition of an entrepreneur.

	W	E	U
W	94.6	2.2	3.2
E	21.6	76.9	1.5
U	56.0	3.7	40.3

Table. 1. Flows between occupations during a year. *Data sources:* authors' computations using CPS data from 2001 to 2015. We restrict our sample to individuals aged between 20 to 65 years old.

B Data: additional elements

In this section, we detail additional elements about our sample selection. The main discussion about data elements is conducted in section 2 and Appendix A of the core paper.

B.1 Part-time versus full-time movements

We display in table 2 the full transition matrix accounting for full and part-time occupations. We class an individual in a part-time occupation when she is working less than 20 hours per week. It appears that individuals working part-time are likely to remain in a part-time occupation (the probability is around 60%). Nonetheless, full-time workers are unlikely to become part-time workers or entrepreneurs whereas full-time entrepreneurs have a chance to become a part-time entrepreneurs. This could be driven, for instance, by a lack of production opportunities forcing entrepreneurs to only work part-time. Overall, the flows of full-time entrepreneurs and workers do not seem to be affected when we distinguish part-time and full-time.

B.2 Not in the labor force (NLF)

In the baseline model, we are computing the flows between three occupations: entrepreneurship, paid-employment and unemployment. We therefore abstract from non-participation. This assumption might create a bias. Table 3 reports the flows between occupations accounting for NLF individuals. There are two main observations. First, entrepreneurial and worker flows are relatively unchanged when we take into account the NLF population. Concerning the unemployment flows, we note that a substantial fraction of unemployed individuals falls into a

	Transition (%)				
	W_F	W_P	E_F	E_P	U
W_F	95.34	2.23	0.39	0.05	1.98
W_P	32.99	61.38	0.48	0.74	4.42
E_F	4.00	0.32	90.44	4.42	0.82
E_P	4.33	3.67	31.34	58.5	2.16
U	39.68	7.67	1.70	0.69	50.25

Table. 2. Flows between occupations for different definitions of entrepreneurship per quarter. *Data sources:* authors' own computations using CPS data from 2001 to 2008.

NLF status for various reasons (discouragement, not actively searching for a job, not directly available for work, etc.). At the same time, an important fraction of the NLF population seems to switch between employment and unemployment. This may indicate that some of them are still attached to the labor force if an opportunity of work becomes available. This could be due to recently graduated young individuals or long-run and discouraged unemployed individuals who finally find a job, without actively looking for it.

	Transition (without NLF)			Transition (with NLF)			
	W	E	U	W	E	U	NLF
W	97.35	0.50	2.15	94.17	0.48	2.08	3.27
E	4.8	94.22	0.99	4.62	90.67	0.95	3.76
U	47.36	2.4	50.25	36.61	1.85	38.84	22.70
NLF	-	-	-	8.50	1.08	3.66	86.77

Table. 3. Flows in percentage between occupations during a quarter, taking into account not in the labor force (NLF) individuals. *Data sources:* flows computed using the monthly basic CPS from 2001 to 2008. We restrict our sample to individuals aged between 20 to 65.

C Detailed model characterization

In this section, we provide detailed value functions characterizations to support the more compact form used in sections 3.3, 3.4 and 3.5 of the core paper. For convenience, we note W the value function associated with a worker, U with an unemployed individual and E an entrepreneur. We characterize here the credit status with the superscript e and the insurance status with the subscript j, except for a worker who is by definition always insured. The future values of those value functions are respectively noted:

$$W^{e\prime} = W(a', \theta', y', e'), \qquad U_j^{e\prime} = U(a', \theta', e', j), \qquad E_j^{e\prime} = E(a', \theta', z', e', j)$$

As with the simplified notations of the model, the continuation value $\mathcal{E}_j^{e\prime}$ defines the future value of a new entrepreneur starting a business with insurance status j and credit status e.

Workers Following the notations of the paper, we can write the value function of a worker in details as follows:

$$
\begin{aligned}
W(a,\theta,y,e) = \max_{c,a',s_e} u(c,0,s_e) + \beta \sum_{\theta'\in\Theta}\sum_{y'\in\mathcal{Y}} \pi(y'|y)\pi(\theta'|\theta) \\
\Big\{(\mathbb{1}_{e=A} + \phi\mathbb{1}_{e=C})\Big[(1-\eta)\Big(\pi_e \max\{W^{A\prime},\mathcal{E}_n^{A\prime}\} + (1-\pi_e)W^{A\prime}\Big) \\
+ \eta\Big(\pi_e \max\{U_i^{A\prime},\mathcal{E}_i^{A\prime}\} + (1-\pi_e)U_i^{A\prime}\Big)\Big] \\
+ (1-\phi)\mathbb{1}_{e=C}\Big[(1-\eta)\Big(\pi_e \max\{W^{C\prime},\mathcal{E}_n^{C\prime}\} + (1-\pi_e)W^{C\prime}\Big) \\
+ \eta\Big(\pi_e \max\{U_i^{C\prime},\mathcal{E}_i^{C\prime}\} + (1-\pi_e)U_i^{C\prime}\Big)\Big]\Big\}
\end{aligned}
$$

$$\text{s.t.} \quad (2),(3),(4)$$

Notice that when $e = A$, then $\pi_c(e' = A|e = A) = 1$. Hence a worker with access to the credit market remains non excluded next period. In the other case, if $e = C$, then $\pi_c(e' = A|e = C) = \phi$. The simplified notations combines those probability in the expectation operator.

Unemployed individual Following the notations of the paper, we can write the value function of an unemployed individual in details as follows:

$$
\begin{aligned}
U(a,\theta,e,j) = \max_{c,a',s_w,s_e} u(c,s_w,s_e) + \beta \sum_{\theta'\in\Theta}\sum_{y'\in\mathcal{Y}} \Pi_y(y')\pi(\theta'|\theta) \\
\Big\{(\mathbb{1}_{e=A} + \phi\mathbb{1}_{e=C})\Big[\pi_w\Big((1-\pi_e)W^{A\prime} + \pi_e\mathcal{U}_j^A(W,E)\Big) \\
+ (1-\pi_w)\Big(\pi_e\mathcal{U}_j^A(U,E) + (1-\pi_e)\mathcal{U}_j^A(U)\Big)\Big] \\
+ (1-\phi)\mathbb{1}_{e=C}\Big[\pi_w\Big((1-\pi_e)W^{C\prime} + \pi_e\mathcal{U}_j^C(W,E)\Big) \\
+ (1-\pi_w)\Big(\pi_e\mathcal{U}_j^C(U,E) + (1-\pi_e)\mathcal{U}_j^C(U)\Big)\Big]\Big\}
\end{aligned}
$$

s.t.

$$
\begin{aligned}
&\mathcal{U}_j^{e\prime}(W,E) = \mathbb{1}_{\{j=i\}}\Big((1-\rho)\max\{W^{e\prime},\mathcal{E}_i^{e\prime}\} + \rho\max\{W^{e\prime},\mathcal{E}_n^{e\prime}\}\Big) + \mathbb{1}_{\{j=n\}}\max\{W^{e\prime},\mathcal{E}_n^{e\prime}\} \\
&\mathcal{U}_j^{e\prime}(U,E) = \mathbb{1}_{\{j=i\}}\Big((1-\rho)\max\{U_i^{e\prime},\mathcal{E}_i^{e\prime}\} + \rho\max\{U_n^{e\prime},\mathcal{E}_n^{e\prime}\}\Big) + \mathbb{1}_{\{j=n\}}\max\{U_n^{e\prime},\mathcal{E}_n^{e\prime}\} \\
&\mathcal{U}_j^{e\prime}(U) = \mathbb{1}_{\{j=i\}}\Big((1-\rho)U_i^{e\prime} + \rho U_n^{e\prime}\Big) + \mathbb{1}_{\{j=i\}}U_n^{e\prime} \\
&(6),(7),(3),(4)
\end{aligned}
$$

With ρ the probability that an unemployed individual loses her UI rights next period. The probability of getting the transitory shock y' is given by the invariant probability distribution Π_y and the shock is known before the decision to take the job.

Non excluded entrepreneur - repayment case

$$R(a,k,\theta,z,j) = \max_{c,a',s_w} u(c,s_w,0) + \beta \sum_{\theta'\in\Theta}\sum_{y'\in\mathcal{Y}} \Pi_y(y')\pi(\theta'|\theta)$$
$$\Big\{(\mathbb{1}_{\{j=i\}}q + \mathbb{1}_{\{j=n\}})\Big(\pi_w \max\{W^{A\prime}, E_n^{A\prime}\} + (1-\pi_w)\max\{U_n^{A\prime}, E_n^{A\prime}\}\Big)$$
$$+ \mathbb{1}_{\{j=i\}}(1-q)\Big(\pi_w \max\{W^{A\prime}, E_i^{A\prime}\} + (1-\pi_w)\max\{U_i^{A\prime}, E_i^{A\prime}\}\Big)\Big\}$$

s.t. (3), (7), (11), (12)

Such an entrepreneur keeps her access to the credit market next period.

Non excluded entrepreneur - bankruptcy case

$$B(a,k,\theta,z,j) = \max_{c,a',s_w} u(c,s_w,0) + \beta \sum_{\theta'\in\Theta}\sum_{y'\in\mathcal{Y}} \Pi_y(y')\pi(\theta'|\theta)$$
$$\Big\{\pi_w W^{C\prime} + (1-\pi_w)\big((\mathbb{1}_{\{j=i\}}q + \mathbb{1}_{\{j=n\}})U_n^{C\prime} + \mathbb{1}_{\{j=i\}}(1-q)U_i^{C\prime}\big)\Big\}$$

s.t. (3), (7), (14), (15)

Such an entrepreneur is excluded from the credit market next period.

Excluded entrepreneur

$$\hat{E}(a,k,\theta,z,j) = \max_{c,a',s_w} u(c,s_w,0) + \beta \sum_{\theta'\in\Theta}\sum_{y'\in\mathcal{Y}} \Pi_y(y')\pi(\theta'|\theta)$$
$$\Big\{(\mathbb{1}_{\{j=i\}}q + \mathbb{1}_{\{j=n\}})\Big((1-\phi)\Big[\pi_w \max\{W^{C\prime}, E_n^{C\prime}\} + (1-\pi_w)\max\{U_n^{C\prime}, E_n^{C\prime}\}\Big]$$
$$+ \phi\Big[\pi_w \max\{W^{A\prime}, E_n^{A\prime}\} + (1-\pi_w)\max\{U_n^{A\prime}, E_n^{A\prime}\}\Big]\Big)$$
$$+ \mathbb{1}_{j=i}(1-q)\Big((1-\phi)\Big[\pi_w \max\{W^{C\prime}, E_i^{C\prime}\} + (1-\pi_w)\max\{U_i^{C\prime}, E_i^{C\prime}\}\Big]$$
$$+ \phi\Big[\pi_w \max\{W^{A\prime}, E_i^{A\prime}\} + (1-\pi_w)\max\{U_i^{A\prime}, E_i^{A\prime}\}\Big]\Big)\Big\}$$

Subject to: (3), (7), (20), (21)

ϕ is the probability to recover the access to the credit market.

D Policy functions and resulting distributions

D.1 Search policy functions

We report below the search policy functions of the workers and the entrepreneurs generated by the baseline model and discussed in section 5 of the core paper.

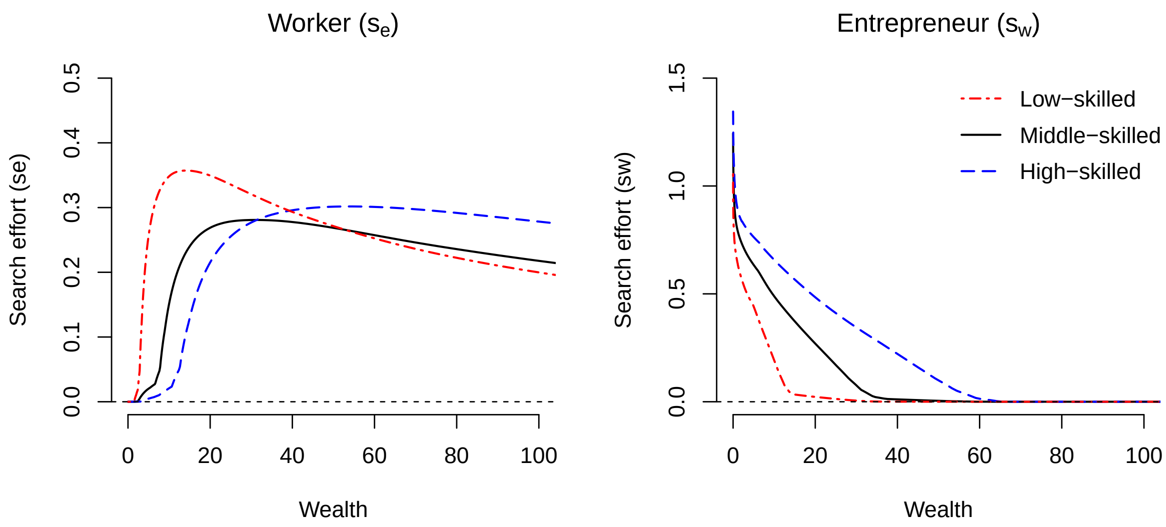

Figure 6. Workers' optimal business search effort (left) and entrepreneurs' optimal job search effort (right). *Note:* these are optimal policies under the non-exclusion case. For workers we choose $y = y_3$ and for entrepreneurs we take $z_{-1} = z_1$.

D.2 Endogenous spreads and borrowing constraints

In the baseline model, an entrepreneur will default only when the business shock z is too small. This is because, in such a case, an entrepreneur generates a small loss and expected future profits are small. Because the external creditor perfectly anticipates this behavior, it charges a higher price to risky entrepreneurs, with a higher incentive to default. The resulting interest rate depends on the entrepreneur's states. In particular, entrepreneurs with sufficient levels of wealth would never default. Indeed, for those entrepreneurs, bankruptcy costs (fees and expected losses from credit market exclusion) are high as compared to the benefits of renegotiating their debt. The incentive to default is also strongly related to the business shock z_{-1} realized during the previous period. Entrepreneurs who experienced a bad shock have a higher probability of default, lowering their ability to borrow by increasing the charged interest rate.

D.3 Distributions

Figure 8 displays the distribution of the three occupations in the model. As in Cagetti and De Nardi (2006), distributions display important concentration of wealth in the hand of entrepreneurs, consistent with the data.

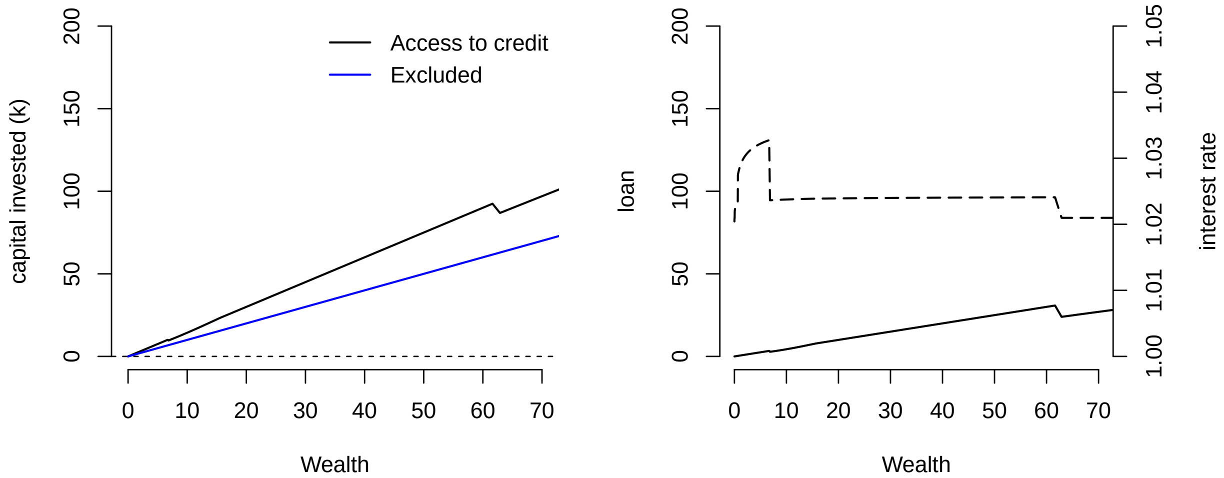

Figure 7. Entrepreneur's policy functions. Left panel: capital invested k, right panel: loan $k - a$ and resulting interest rate $r(\Phi, k)$ for non-excluded entrepreneurs. Plot correspond to $\theta = \theta_2$ and $z = z_6$.

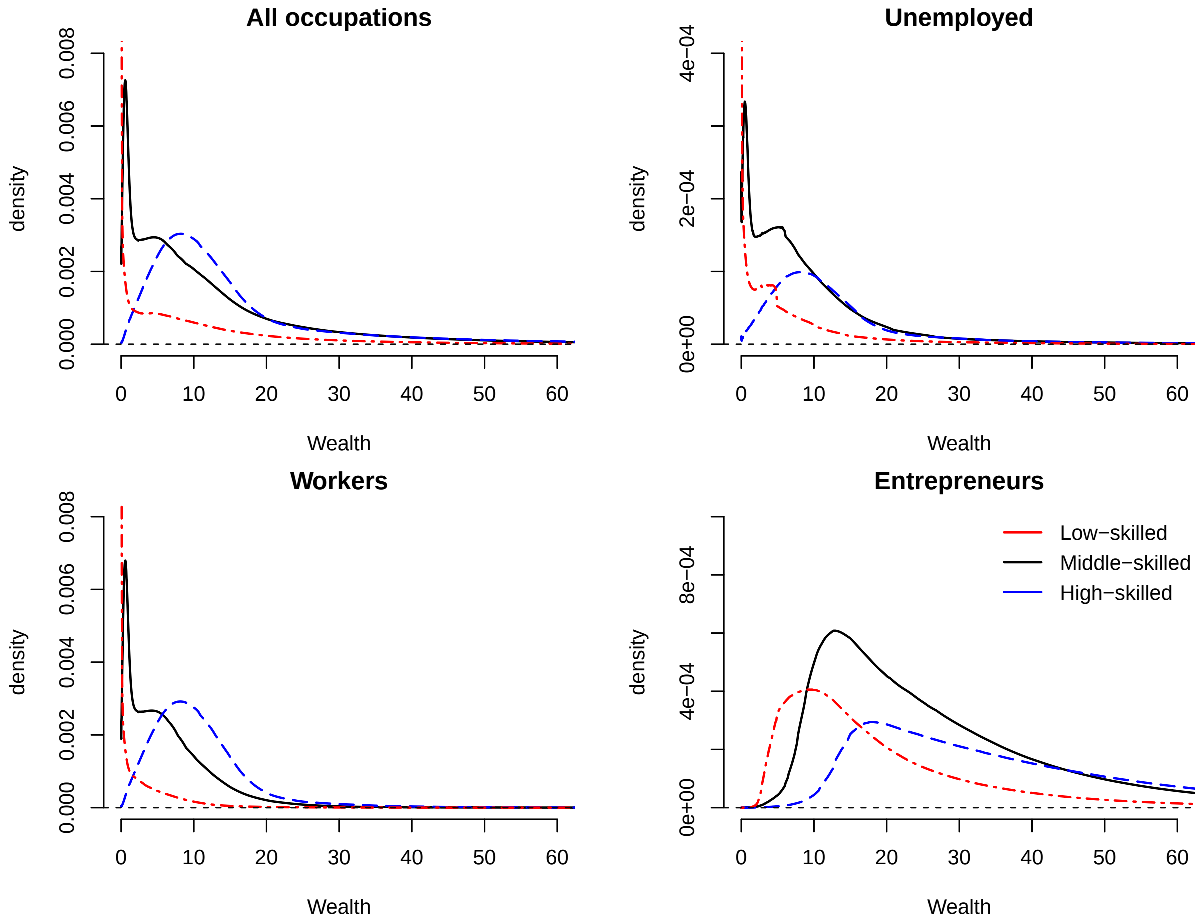

Figure 8. Distribution for the three occupations.

E Transitional dynamics and welfare

In this section, we discuss in greater details the transitional dynamics and the welfare outcomes of the different policies considered in the main paper. The related discussion in the paper is in section 6.6.

E.1 Transitional dynamics and convergence to a new steady-state

In this sub-section, we study the dynamic effects of suddenly, and unexpectedly, introducing the policies detailed in the core of the paper. Along the perfect foresight transitional path that ends at the new steady-state, all agents correctly forecast the sequence of prices, and markets clearing in each period. We assume that at $t = 0$, the economy is at the initial steady-state characterized by the absence of any policy. At $t = 1$ the reform (either the DRI or the SUS) is implemented and is unanticipated by agents. The economy then converges to the new steady-state[1].

In Figure 10, we depict the transitional dynamics of the economy after an unexpected introduction of either the DRI or the SUS[2]. Under the two reforms, the share of entrepreneurs sharply increases (by 0.25% under the DRI and 1% under the SUS) and then smoothly converges to the new steady-state level. The convergence is slower under the DRI, since the fraction of unemployed individuals starting businesses is smaller. After 5 years, 84% of the occupational adjustment has taken place under the SUS and 47% under the DRI.

Regarding other occupations, the adjustment is similar for corporate jobs. Under the DRI, self-employment crowd out corporate jobs slowly after a sharp decline in the first period, since unemployed individuals redirect their search toward finding a business idea. This crowding-out effect is smaller under the SUS. Instead, the unemployment rate sharply declines by 0.8% and then stays fairly stable along the transition. Consequently, under the DRI, the first increase in the share of entrepreneurs is driven by a shift of the occupational choice (extensive margin) of unemployed individuals, that have the opportunity to either open a business or work in a corporate job. Contrastingly, the increase under the SUS is due to a fraction of unemployed individuals that find it now more interesting to start a business than staying unemployed. Those individuals increase their search effort to find a business idea. Taken together, the results show that much of the response of household occupational shift occurs on the extensive margin under the DRI while it comes from an intensive margin under the SUS.

[1]To solve the transition, we compute the solutions of the household problem backward, starting at the new steady-state. We then find prices that are consistent with the implied policies and we iterate until convergence. Details of the full algorithm is provided in the online appendix.

[2]This assumption of an unexpected introduction of the policies is standard in the incomplete markets literature. Of course, the policies could be in practice announced before their implementation and therefore anticipated by agents.

Concerning the financing of the policies along the transition, the tax rate sharply increases directly after their implementation to its steady-state level under the SUS, while it is smoothed with the DRI as the number of entrepreneurs within the program increases.

On the aggregate capital stock, corporate investment decreases under the two policies after implementation. This lower capital invested in the corporate sector is in fact compensated by a higher level of capital invested in the entrepreneurial sector. Under the SUS, the corporate capital adjustment changes and increases after 3 years. Since the failing and exit rate under this policy are higher, entrepreneurs that entered the program tend to stop their businesses and invest their wealth in the corporate sector either voluntarily or involuntarily. Production is only slightly impacted by the reforms. Under the two reforms, the production of the entrepreneurial sector increases and overcomes the decline of the corporate sector.

E.2 Welfare analysis

Following Flodén (2001), we compute the welfare change between steady-states in consumption equivalent variations. Our results suggest that the welfare gain of a newborn agent who is dropped at random in the distribution (i.e. the *ex-ante* welfare measure) is positive under the two reforms. In terms of consumption equivalent variation (cev), the welfare increases by 0.084% under the SUS policy and by 0.02% under the DRI policy. Taking into account the transitional dynamics, it respectively increases by 0.059% and by 0.057%. Concerning the conditional welfare changes (at every point of the state space), it appears that the DRI benefits mostly (and largely) entrepreneurs who enter the program. All other occupations are slightly worse off due to the increased tax rate imposed on workers. Poor individuals are those with the highest welfare reduction since they are too constrained to start a business, even under the DRI. On the contrary, the SUS seems to be more beneficial between steady-states for the vast majority of the individuals. In particular, it benefits mostly poor individuals. Figure 9 shows the *ex-ante* welfare change for insured unemployed individuals along the transition.

The reported welfare gains and costs are small, but we note that the policies concern only a very small fraction of the economy while its cost is supported by the majority. Indeed, the gains that the policies produce are partially reduced because of the way the policies are financed. As we considered the policies as a UI extension plan, the source of financing is a labor income tax on every worker. We surmise that had the financing been different, for instance by taxing the current pool of entrepreneurs, the welfare results might have been different.

E.3 Welfare computation

We use the consumption equivalent variation as defined by Flodén (2001), in order to compute the welfare along the transition and between the two steady-states. The conditional welfare

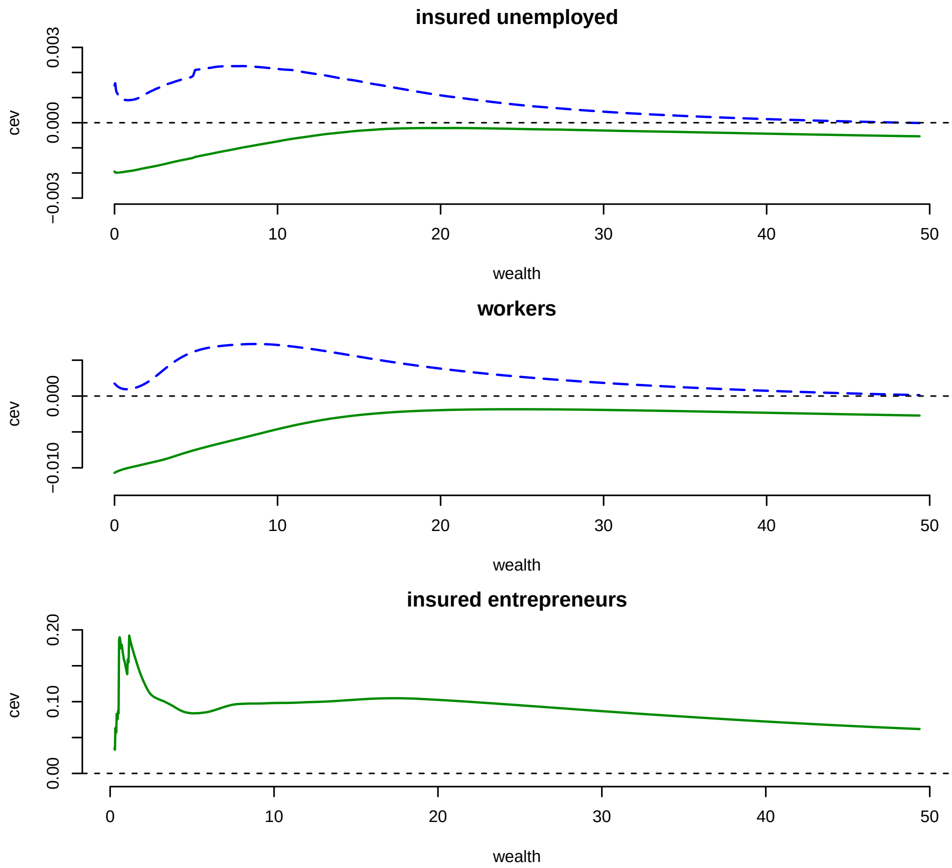

Figure 9. Conditional welfare change (in *cev*) for different occupation between steady states. *Note:* the solid green line refers to DRI and the dashed blue line to SUS.

change, $\omega(\mathbf{x})$, for a household with state vector $\mathbf{x}$ of a policy change between an economy A to another economy B is defined by:

$$E_0 \sum_{t=0}^{\infty} u(c_t^A(1+\omega(\mathbf{x})), s_{w,t}^A, s_{e,t}^A) = E_0 \sum_{t=0}^{\infty} u(c_t^B, s_{w,t}^B, s_{e,t}^B)$$

where in our case $\omega(\mathbf{x})$ is given by:

$$\omega(\mathbf{x}) = \left(\frac{E_0 \sum_{t=0}^{\infty} u(c_t^B, s_{w,t}^B, s_{e,t}^B)}{E_0 \sum_{t=0}^{\infty} u(c_t^A, s_{w,t}^A, s_{e,t}^A)}\right)^{\frac{1}{1-\sigma}} - 1$$

We also compute the *ex-ante* utilitarian social welfare change, by computing the premium ω_V that measures the percent of life-time consumption that a newborn in the economy A would need to be indifferent between economy A (without the policy) and B (with the policy). Following Flodén (2001), it can be shown that in the case of a CRRA utility with risk-aversion parameter σ, we have:

$$\omega_V = \left(\frac{V^B}{V^A}\right)^{\frac{1}{1-\sigma}} - 1$$

where the life-time utility over all households in the economy i, V^i, is defined as:

$$V^i = \int_{\mathbf{x}} E_0 \sum_{t=0}^{\infty} u(c_t^i, s_{w,t}^i, s_{e,t}^i) d\Gamma^i(\mathbf{x})$$

Along the transition, we proceed in a similar fashion. The economy A is defined as the economy in period t whereas the economy B is the next period economy in period $t+1$. This allows us to assess the effect of the policy experiments along the transitional path. It measures the constant increment in percentage of consumption in every state that has to be given to each agent so that she is indifferent between remaining in the benchmark economy and moving to another economy that makes a transition to a new steady-state implied by the reforms.

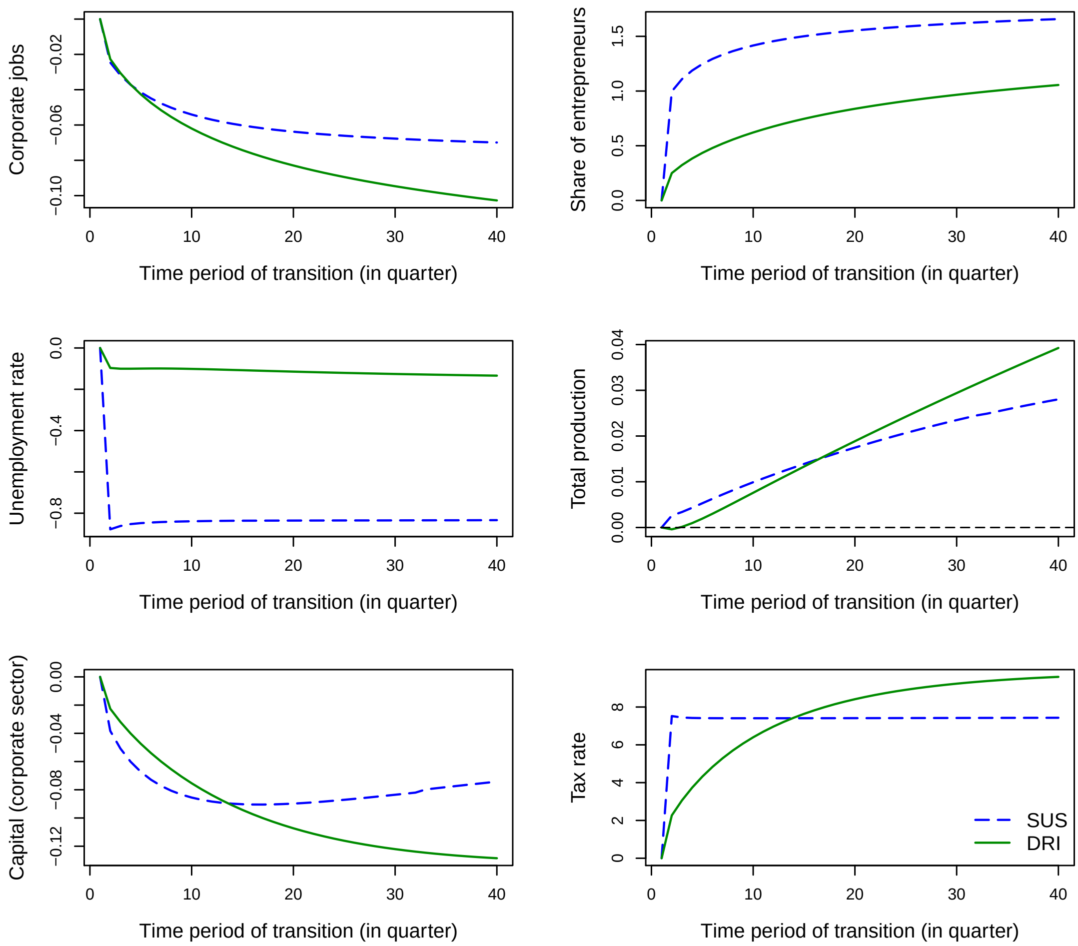

Figure 10. Transitional dynamics of the Economy after an unexpected introduction of the policies at t = 1. *Note:* the solid green line refers to the DRI policy and the dashed blue line to the SUS experiment.

F Numerical implementation

State space and grid definition In our model, an household is fully characterized by a state vector $\mathbf{x} = (o, y, \theta, z, e, j, a)$ with $a \in A$, $y \in \mathcal{Y}$, $z \in \mathcal{Z}$, $\theta \in \Theta$, $o \in \{w, e, u\}$, $e \in \{A, C\}$ and $j \in \{i, n\}$. We compute the household problem using a grid of asset $\mathbf{a}$ of 350 points, spaced according to an exponential rule. We discretize the process z, y and θ with respectively 7, 5 and 3 grid points. We compute the second stage entrepreneur's problem over a grid of cash-on-hand with 350 grid points.

F.1 Algorithm

We organize the algorithm as follows.

1. Initialize a full dimension grid space composed of all different possible asset values (a), productivity level (y), innate ability (θ) and entrepreneurial state (z). The maximum asset level is chosen sufficiently large to get ergodicity of the policy functions.

2. Guess initial tax rate τ_w and prices $\{w, r\}$.

3. Given prices, solve the consumption-saving-search (CSS) problem of a worker and an unemployed agent.

4. For the entrepreneur's problem, we proceed as follows.

   - First we solve the CSS problem of the values B, R and $\hat{E}$ on a grid of cash-on-hand.
   - Given the solution to the previous values, set a grid of possible investment value k with bound $[0, \lambda a]$.
   - Separate the problem in multiple regions. Between $[0, a]$, we apply a standard solver to find the optimal k. Between $[a, \lambda a]$, we apply a grid search that account for multiple solutions that could arise due to the endogenous determination of the spread r^b.
   - For each $k > a$, start by providing the loan at the risk-free interest rate r. If the entrepreneur default for this interest rate, then compute the resulting new interest rate r^b implied by the zero profit condition of the bank. Iterate the process until r^b is consistent with the default probability. A loan that implies a default probability equals to 1 is not allowed.
   - Save the best solutions to the problem and find the optimal k level.

5. Construct the transition matrix $\mathbf{M}$ generated by Π_y, Π_z and Π_θ, $a'(\mathbf{x})$, $s_w(\mathbf{x})$, $s_e(\mathbf{x})$ and the default decision. Compute the associated stationary measure of individuals $\Gamma(\mathbf{x})$, by first guessing an initial mass of one of households with zero asset and then by iterating on $\Gamma'(\mathbf{x}) = \mathbf{M}\Gamma(\mathbf{x})$ until $|\Gamma'(\mathbf{x}) - \Gamma(\mathbf{x})| < \mu$, with μ very small.

6. Compute the resulting total asset level, total labor supplied and total investment in the entrepreneurial sector. Total capital invested in the corporate sector is given as the difference between total savings and total capital invested in the entrepreneurial sector. Total labor used in the corporate sector is given by total labor supplied by workers.

7. Update prices $\{r, w\}$ using the marginal productivities in the corporate sector and tax rate τ_w to close the government budget up to a relaxation. Back to step 2 until convergence of labor income tax rate and prices.

F.2 Optimal search efforts

We describe here the solution algorithm for computing ex-ante all the optimal search efforts. Given a set of parameter $(\kappa_w, \kappa_e, \psi_e, \psi_w)$, the solutions s_w and s_e for each occupation is computed only once.

In order to pre-compute the search efforts, we set up a very large grid that we call **diffval** ($\mathcal{G}_{\mathbf{\Delta V}} = [0, dmax]$), which summarizes the option values (ΔV) of interest, that are needed to compute either s_w or s_e, as shown above. Given this grid, we solve for the optimal search efforts. We end up with grid $\mathcal{G}_{\mathbf{w}}$ and $\mathcal{G}_{\mathbf{e}}$ over optimal search efforts corresponding to values in the grid $\mathcal{G}_{\mathbf{\Delta V}}$. When solving for the household problem, we therefore compute ΔV and we find, using linear interpolation, the corresponding optimal search efforts $s_w(\Delta V)$ and $s_e(\Delta V)$.

Worker and entrepreneur search efforts The solution for the optimal search efforts of a worker and an entrepreneur (who repays) is straightforward and are given respectively by the first order conditions:

$$\frac{\partial W(a,\theta,y,e)}{\partial s_e} = 0$$

$$-\psi_w(s_w)^{\psi_w} + \beta\pi'_w(s_w)\underbrace{\Big[\eta \max\{0, \mathcal{E}'_i - U'_i\} + (1-\eta)\max\{0, \mathcal{E}'_n - W'\}\Big]}_{\Delta V > 0} = 0$$

$$\frac{\partial R(a,k,\theta,z,j)}{\partial s_w} = 0$$

$$-\psi_e(s_e)^{\psi_e} + \beta\pi'_e(s_e)\underbrace{\Big[\max\{W', E'_{j'}\} - \max\{U'_{j'}, E'_{j'}\}\Big]}_{\Delta V > 0} = 0$$

Unemployed individuals An individual who is currently unemployed can search at the same time a business idea and a job. A convenient way to rewrite the value function in order to solve ex-ante the optimal search efforts is to use option values as follows:

$$\begin{aligned} U(a,\theta,e,j) = \max_{c,a',s_w,s_e} & \; u(c, s_w, s_e) \\ & + \beta\mathbb{E}_{\theta',y',j',e'|\theta,y,j,e}\Big\{U'_{j'} + \pi_w(W' - U'_{j'}) + \pi_w\pi_e\max\{0, \mathcal{E}'_{j'} - W'\} \\ & + (1-\pi_w)\pi_e\max\{0, \mathcal{E}'_{j'} - U'_{j'}\}\Big\} \end{aligned}$$

The job search effort solves:

$$\frac{\partial U(a,\theta,e,j)}{\partial s_w} = 0$$

$$u_{s_w} + \beta\Big[\pi'_w(W' - U'_{j'}) + \pi'_w\pi_e \max\{0, \mathcal{E}'_{j'} - W'\} - \pi'_w\pi_e \max\{0, \mathcal{E}'_{j'} - U'_{j'}\}\Big] = 0$$

and the condition for the business search effort is:

$$\frac{\partial U(a,\theta,e,j)}{\partial s_e} = 0$$

$$u_{s_e} + \beta\Big[\pi_w\pi'_e \max\{0, \mathcal{E}'_{j'} - W'\} + (1-\pi_w)\pi'_e \max\{0, \mathcal{E}'_{j'} - U'_{j'}\}\Big] = 0$$

Using the notation $\mathcal{P}(s_w) = \pi_w \max\{0, \mathcal{E}'_{j'} - W'\} + (1-\pi_w)\max\{0, \mathcal{E}'_{j'} - U'_{j'}\}$, we get the following condition for the optimal business search effort:

$$\pi_e(s_e^*) = 1 - \frac{\psi_e(s_e^*)^{\psi_e}}{\beta\kappa_e\mathcal{P}(\bar{s}_w)}$$

At the optimal search effort, the probability of finding a business idea $\pi_e(s_e^*)$ is decreasing with the cost of the search $\psi_e(s_e^*)^{\psi_e}$ and increasing with the discount factor β, the matching parameter κ_e and the value associated of being entrepreneur relative to other occupations $\mathcal{P}(\bar{s}_w)$. We pre-compute the optimal search effort s_e^* given ΔV and s_w. We then apply a root-finding optimizer to search for the corresponding s_w^* within the CSS problem.

F.3 Transitional dynamics

We assume that the economy is in the initial steady state in period 0 and the reform is announced and implemented in period 1. Agents did not anticipate the policy before its implementation. The economy makes a transition to reach the final steady state in period T. We choose T large enough so that the resulting stationary distribution in period T is close enough to the post-reform steady states. The algorithm for transition dynamics is as follows:

1. Guess a path for $\{\mathcal{L}_1, ..., \mathcal{L}_{T-1}\}$ with $\mathcal{L}_t = \{r_t, w_t, \tau_{w,t}\}$. $\mathcal{L}_0$ and $\mathcal{L}_T$ are given by initial and final steady-states.

2. Use value functions of the final steady state (period T) to solve the households' problem backwards starting from $T-1$ until period 1.

3. Use the distribution of the initial steady state and the resulting policy functions to compute the path of the distribution of household $\{\hat{\Gamma}(\mathbf{x})_1, ..., \hat{\Gamma}(\mathbf{x})_T\}$.

4. Given these distributions, compute new path $\{\mathcal{L}_1, ..., \mathcal{L}_{T-1}\}$. Iterate from step 2 until the difference between the initial path is close enough to the resulting path.

5. When convergence is achieved, check if the resulting final distribution $\hat{\Gamma}(\mathbf{x})_T$ is close enough to the steady-state distribution $\Gamma(\mathbf{x})_T$ up to a relaxation. If the two distributions are identical, then stop, else, increase the number of periods T.

G Robustness

Reparameterization using self-employment We have parameterized again the model using self-employment as the definition for entrepreneurship and assessed the impact of the DRI in such an economy. Table 4 summarizes the parameters and the moments of the new calibration. We target a higher exit rate and a larger fraction of entrepreneurs. We also target a different shape of the transition from paid-employment to entrepreneurship, in line with the data.

A. Parameter	**Symbol**	**Value**
Discount factor	β	0.9742
z process (autocorrelation, variance)	ρ_z, σ_z^2	0.8636, 0.197
Businesses' return to scale	ν	0.79
Search utility parameter	$\psi_e = \psi_w$	2.579
Matching parameter	κ_e,κ_w	0.328,0.842
Bankruptcy cost	ϕ	0.0275
Entrepreneur's innate ability	$[g_1, g_2, g_3]$	[0.0686, 0.0772, 0.1007]
B. Targeted moments	**Target**	**Model**
Unemployment rate (in %)	5.0	5.0
Share of entrepreneurs (in %)	10.5	10.6
Entrepreneurs' exit rate (in %)	6.5 - 7.5	7.1
Fraction of new entrepreneurs prev. unemployed (in %)	20	20
Capital-output ratio (annual)	2.65	2.66
Ratio of net worth E/pop	7 - 8	7.6
Bankruptcy rate (as fraction of entrepreneurs) (in %)	0.57	0.58
Fraction of entrepreneur with neg. earnings (in %)	10	10.5
Flows W to E by quantiles / avg rate (%)	$\begin{bmatrix} Q1 & Q2 & Q3 \\ 1.1 & 0.85 & 1.0 \end{bmatrix}$	$\begin{bmatrix} Q1 & Q2 & Q3 \\ 1.11 & 0.89 & 1.0 \end{bmatrix}$

Table. 4. Estimated parameters and targeted moments for the alternative model with self-employment as entrepreneur's definition.

Again, the model is able to reproduce the main features concerning the observed transitions. Table 5 summarizes the aggregate transitions in the model against the data. The magnitudes found in the model are close to their data counterparts.

In table 6, we summarize the results of the model under the two considered reforms: DRI and SUS. Results are quite similar to our baseline model but effects are stronger with all self-employed. Most notably, the share of entrepreneurs increases by more than 2.1% and the share of insured entrepreneurs account for 3.3% of all entrepreneurs under the DRI in this alternative specification, against 1.8% and 2.9% when considering self-employed business owners. Finally, we notice that the cost of the SUS is higher than the DRI when considering all self-employed entrepreneurs.

	Mass (%)		Transition: Model (Data) (%)		
	Target	Model	W	E	U
W	84.5	84.4	97.15 (97.20)	0.76 (0.70)	2.09 (2.11)
E	10.5	10.6	6.09 (6.15)	92.93 (92.46)	0.98 (1.40)
U	5.0	5.0	43.88 (46.04)	3.22 (3.72)	52.90 (50.25)

Table. 5. Transition between occupations during a quarter (data counterparts between braces). *Data sources:* authors' computations using CPS data from 2001 to 2008. We restrict our sample to individuals aged between 20 to 65 years old.

	Baseline	DRI	SUS
		($f = 0.3, \bar{q} = 0.5$)	($S = 0.07$)
Fraction of entrepreneurs	10.57	10.79	10.79
% unemployed starting businesses (in %)	3.22	3.59	5.67
entrepreneurship exit rate (in %)	7.07	7.16	8.18
unemployment rate (in %)	4.988	4.974	4.927
corporate jobs (in %)	84.44	84.23	84.28
new firm per year (th.)	500	515.78	585.41
necessity share (in %)	8.5	6.7	11.9
Entrepreneurial sector production	0.386	0.393	0.388
Corporate sector capital	3.58	3.574	3.576
Entrepreneurial sector capital	1.729	1.761	1.734
tax rate τ_w (in %)	0.896	1.019	1.022
Cost of the policy over total production (in %)	-	0.017	0.019

Table. 6. Downside risk insurance (DRI) and start-up subsidy (SUS) effects on aggregates.

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