Questions and Answers *

Alexandre Gaillard
TOULOUSE SCHOOL OF ECONOMICS (M2 ETE)

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Here all (I hope) detailled answers to your questions and doubts during the sessions. If you have other questions, contact me.

1 Session 1

Recall: When time is discrete and time period is 1, growth rate is $\gamma_k = \frac{\partial log(k_t)}{\partial k_t} = \frac{log(k_{t+1}) - log(k_t)}{1} = log(\frac{k_{t+1}}{k_t})$. As for small a, we have $log(a) \approx a - 1$, then $\gamma_k = \frac{k_{t+1}}{k_t} - 1$.

Alternative answer for P3 Q3 Consider $\dot{K}_t = K_{t+1} - K_t$. Divide by the population mass, you get: $\dot{k}_t = K_{t+1}/L_{t+1} - K_t/L_t$. Knowning that $L_{t+1} = (1+n)L_t$ and replacing K_{t+1} by $sY_t + (1-\delta)K_t$ we get:

$$\dot{k}_t = \frac{sy_t + (1-\delta)k_t}{1+n} - k_t = \frac{sy_t - (\delta + n)k_t}{1+n}$$

With the fact that $y_t = k_t$ if $\alpha + \beta = 1$, then

$$\dot{k}_t = \frac{k_t(s - (\delta + n))}{1+n}$$

Finally, note that $\gamma_k = \dot{k}_t/k_t$, thus we also find without log that:

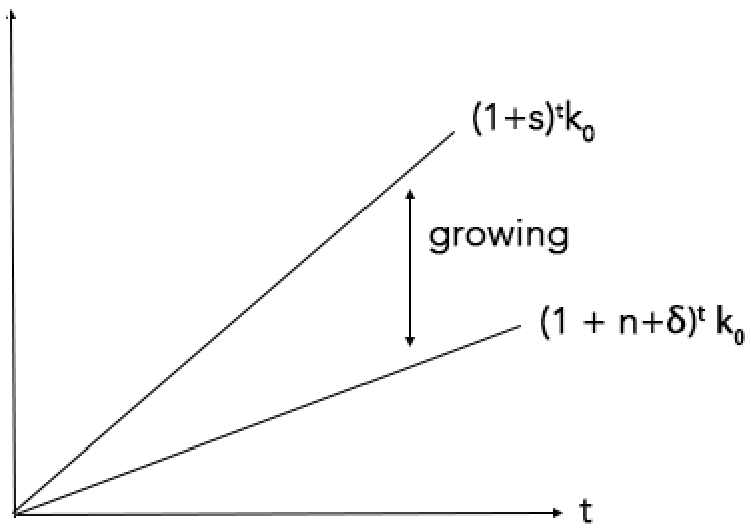
$$\gamma_k = \frac{\dot{k}_t}{k_t} = \frac{s - (\delta + n)}{1+n}$$

Uniqueness before stability? You can have more than one stable fixed point.

Steady state: confusion Q3 - Q4 In question 3, we found $\gamma_k = \frac{s+1-\delta}{1+n} - 1 = \frac{s-(n+\delta)}{1+n}$. A constant growth rate implies that $s > n + \delta$ such that $\{k_t\}_{t=0}^{\infty}$ is unbounded. **In that case: there is no steady state value for** k_t (there is no value of k_t such that $\Delta k_{t+1} = 0$ except if $k_t = 0$).

*Corresponding author: alexandre.gaillard@tse-fr.eu.

Fig. 1. case where $\alpha + \beta = 1$



If $s < n + \delta$, then $\gamma_k < 0$ and the economy **converges to a steady state** $\bar{k} = 0$ (which is stable).

In question 4, we found two possible equilibriums, $k_t^1 = 0$ and k_t^2, but we do not restrict on the case $\alpha + \beta = 1$. The fact that we found two steady states comes from the fact that the "investment" curve cross one time the depreciation line.
Recall: balanced growth path is an economy where variables grow at a constant rate, here K_t and L_t are at a balanced growth path whereas k_t is a steady-state (in volume, it does not move).

Finally, bellow figures which summarize all of this:

Homogeneity of the value function How do we get $v(\alpha A, \alpha Y) = \alpha^{1-\theta} v(A, Y)$ from

$$v(\alpha A, \alpha Y) = \max_{0 \leq \tilde{A}' \leq RA+Y} \frac{\alpha^{1-\theta}}{1-\theta}(A + Y/R - \tilde{A}'/R)^{1-\theta} + \beta v(\alpha \tilde{A}', \mu \alpha Y)$$

One answer could be to use a guess and verify approach. If we suppose $v(\alpha A, \alpha Y) = \alpha^{1-\theta} v(A, Y)$, then:

$$\alpha^{1-\theta} v(A, Y) = \max_{0 \leq \tilde{A}' \leq RA+Y} \frac{\alpha^{1-\theta}}{1-\theta}(A + Y/R - \tilde{A}'/R)^{1-\theta} + \beta \alpha^{1-\theta} v(\tilde{A}', \mu Y)$$

Dividing everything by $\alpha^{1-\theta}$ yields

$$v(A, Y) = \max_{0 \leq \tilde{A}' \leq RA+Y} \frac{1}{1-\theta}(A + Y/R - \tilde{A}'/R)^{1-\theta} + \beta v(\tilde{A}', \mu Y)$$

So that it is indeed homogeneous of degree $1 - \theta$.

Fig. 2. case where $\alpha + \beta > 1$ or < 1